School Of Math

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MATHEMATICS

CLASS XII (Set/A) **COAD : 13E /17**

Time: 3 hours **MM: 100**

General Instructions:

1. *All questions are compulsory.*
2. *The question paper consists of **29** questions divided into three sections **A, B,C** and **D**. Section **A** comprises **4** questions of **one mark** each, Section **B** comprises **8** questions of **two marks** each , Section **C** comprises 11 questions of **four marks** each and Section **D** comprises 6 questions of **six marks** each.*
3. *All questions in Section **A** are to be answered in one word, one sentence or as per the exact requirement of the questions.*
4. *Use of calculator is not permitted. You may ask for logarithmic tables, if required.*

SECTION – A

Q1 Find the value of sec $\left(\tan^{-1}\frac{y}{2}\right)$. 1

Q2 The area of a triangle with vertices (-3,0), (3,0) and (0,k) is 9 sq. units. Find the value of k . 1

Q3 Find the position vector of the point which divides the join of points with position vectors $\vec{a}+\vec{b}$ and $2\vec{a}-\vec{b}$ in the ratio 1 :2 . 1

Q4 Set A has 3 elements and the set B has 4 elements. Then the number of injective mappings that can be defined from A to B . 1

SECTION – B

Q5 In the set N of natural numbers, define the binary operation *by m*n= g.c.d (m,n), m,n $\in N$. Is the operation *commutative and associative? 2

Q6 Without expanding, Evaluate 2

$$\Delta = \begin{vmatrix} \cos ec^2\theta & \cot^2\theta & 1 \\ \cot^2\theta & \cos ec^2\theta & -1 \\ 42 & 40 & 2 \end{vmatrix}$$

Q7 Verify mean value theorem for the function f(x) = (x – 3) (x – 6) (x – 9) in [3,5]. 2

Q8 State whether the following statements are true or false (Give reason). 2

x + y = $\tan^{-1}$ y is a solution of the differential equation $y^2\frac{dy}{dx}+y^2+1=0$

Q9 P is a point on the line segment joining the points (3,2,-1) and (6, 2,-2) . If x co – ordinate of P is 5, then find its y co- ordinate . 2

Q10 The corner points of the feasible region determined by the system of linear constraints are (0,10), (5,5),(15,15),(0,20). Let Z = px + qy, where p,q >0. Condition on p and q so that the maximum of Z occurs at both the points (15,15) and (0,20) . 2

Q11 A and B are two candidates seeking admission in a college. The probability that A is selected is 0.7 and the probability that exactly one of them is selected is 0.6. Find the probability that B is selected. 2

Q12 For the curve y = 5x – 2x^3 , if x increases at the rate of 2 units/ sec, then how fast is the slope of curve changing when x = 3? 2

SECTION – C

Q13 x,y $\in R$ and the determinant $\Delta = \begin{vmatrix} \cos x & -\sin x & 1 \\ \sin x & \cos x & 1 \\ \cos(x+y) & -\sin(x+y) & 0 \end{vmatrix}$ in the interval If $\Delta \in [a,b]$ find a and b . 4

OR

If the determinant $\Delta = \begin{vmatrix} 3 & -2 & \sin 3\theta \\ -7 & 8 & \cos 2\theta \\ -11 & 14 & 2 \end{vmatrix} = 0$ then find $\sin\theta$.

Q14 Discuss the applicability of Rolle's theorem on the function given by $f(x) = \begin{cases} x^2+1, & if\, 0 \le x \le 1 \\ 3-x, & 1 \le x \le 2 \end{cases}$. on [0,2]. 4

Q15 Find the condition for the curves $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1; xy = c^2$ to the intersect orthogonally. 4

OR

Water is dripping out at a steady rate of 1 cu cm/sec through a tiny hole at the vertex of the conical vessel, whose axis is vertical. When the slant height of water in the vessel is 4 cm, find the rate of decrease of slant height, where the vertical angle of the conical vessel is $\frac{\pi}{6}$.

Q16 Evaluate $\int_{-1}^{2} (7x-5)\,dx$ as a limit of sums . 4

Q17 using integration find the area enclosed by the curve x = 3 cost, y = 2 sin t. 4

Q18 Find the equation of a curve passing through $\left(1, \frac{\pi}{4}\right)$ if the slope of the tangent to the curve at any point P (x,y) is $\frac{y}{x} - \cos^2 \frac{y}{x}$. 4

OR

Solve $x^2 \frac{dy}{dx} - xy = 1 + \cos\left(\frac{y}{x}\right), x \neq 0 and x = 1, y = \frac{\pi}{2}$

Q19 If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k} and \vec{b} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, find a vector of magnitude $\sqrt{140}$ units which is coplanar with $\vec{a}$ *and* $\vec{b}$ perpendicular to $\vec{a}$. 4

Q20 Find the co- ordinates of the foot of perpendicular drawn from the point A (1,8,4) to the line joining the points B(0,-1,3) and C (2, -3, -1). 4

Q21 A company manufactures two types of sweaters : type A and type B. It costs Rs 360 to make a type A sweater and Rs 120 to make a type B sweater. The company can make at most 300 sweaters and spend at most Rs 72000 a day. The number of sweaters of type B cannot exceed the number of sweaters of type A by more than 100. The company makes a profit of Rs 200 for each sweater of type A and Rs 120 for every sweater of type B. 4
Formulate this problem as LPP in order that the profit is maximum.

Q22 For a loaded die, the probabilities of outcomes are given as under: P(1) = P(2) = 0.2, P(3)=P(5)=P(6) = 0.1 and p(4) = 0.3. 4
The die is thrown two times. Let A and B be the events, ‘same number each time’ , and ‘a total score is 10 or more’, respectively. Determine whether or not A and B are independent.

Q23 A committee of 4 students is selected at random from a group consisting 8 boys and 4 girls. Given that there is at least one girl on the committee, calculate the probability that there are exactly 2 girls on the committee. 4

SECTION – D

Q24 Evaluate : $\int_{-2}^{2} \left|x\cos\pi x\right| dx$. 6

OR

Find $\int_{0}^{1} x\left(\tan^{-1}x\right)^{2} dx$

Q25 Show that : $2\tan^{-1}\left\{\tan\frac{\alpha}{2}.\tan\left(\frac{\pi}{4}-\frac{\beta}{2}\right)\right\}=\tan^{-1}\frac{\sin\alpha\cos\beta}{\cos\alpha+\sin\beta}$ 6

OR

Relation S in the set A = $\{x\in Z; 0\le x\le 12\}$ given by $S=\{(a,b): a,b\in Z, |a-b|$ is divisible by 4}. Show that S is an equivalence relation. Find the equivalence class of 1.

Q26 Find x, y, z if A = $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies A’ = A^{-1}. 6

Q27 Examine the differentiability of the function f defined by $f(x)=\begin{cases} 2x+3 & if \quad -3\le x<-2 \\ x+1 & if \quad -2\le x<0 \\ x+2 & if \quad 0\le x\le 1 \end{cases}$. 6

Q28 The sum of the surface areas of a rectangular parallelepiped with sides x, 2x and $\frac{x}{3}$ and a sphere is given to be constant. Prove that the sum of their volumes is minimum, if x is equal to three times the radius of the sphere. Also find the minimum value of the sum of their volumes. 6

Q29 Find the image of the point having position vector $\hat{i}+3\hat{j}+4\hat{k}$ in the plane $\hat{r}.\left(2\hat{i}-\hat{j}+\hat{k}\right)+3=0$. 6

Ans : 1 $\sec\theta = \frac{\sqrt{4+y^2}}{2}$ 2 3 3 $\frac{4\vec{a}+\vec{b}}{3}$ 4 24 5 $(l*m)*n.$ 6 $\Delta = \begin{vmatrix} 0 & \cot^2\theta & 1 \\ 0 & \operatorname{cosec}^2\theta & -1 \\ 0 & 40 & 2 \end{vmatrix}$

8 $\frac{-(1+y^2)}{y^2}$ 9 2 10 q = 3p 11 p = 0.25 12 -72 units / sec 13 a = $-\sqrt{2}, b=\sqrt{2}$

14 $\sin\theta = \frac{1}{2}$ 15 $a^2-b^2=0$ or $\frac{1}{2\sqrt{3}\pi} cm/s$ 16 $\frac{-9}{2}$ 17 $6\pi sq\ units$

18 $\tan\left(\frac{y}{x}\right)+\log x = 1$ or $k=\frac{3}{2}, \tan\left(\frac{y}{2x}\right) = -\frac{1}{2x^2}+\frac{3}{2}$

19 $\vec{p} = \pm 2\left(\hat{i}+3\hat{j}-5\hat{k}\right)$ 20 $\left(\frac{-5}{3}, \frac{2}{3}, \frac{19}{3}\right)$

21 Maximise Z = 200 x + 120y subject to : $x+y \le 300, 3x+y \le 600, y \le x+100, x \ge 0, y \ge 0$

22 independent 23 $\frac{168}{425}$ 24 $\frac{8}{\pi}$ or $\frac{\pi^2-4\pi}{16}+\log\sqrt{2}$ 26 $x = \pm\frac{1}{\sqrt{2}}, y=\pm\frac{1}{\sqrt{6}}, z = \pm\frac{1}{\sqrt{3}}$

27 f is not differentiable at x = 0 28 $\frac{2}{3}x^3\left(1+\frac{2\pi}{27}\right)$

29 $.-3\hat{i}+5\hat{j}+2\hat{k}$.